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BRIDGING THE GAP IN MATHEMATICAL ABILITY OF MTs STUDENTS: AN EXPERIMENTAL STUDY OF THE KNISLEY MODEL BASED ON TANGRAM MEDIA

Sarach Nur Wahidah[1], Andhita Dessy Wulansari[2]
[1.2]Universitas Islam Negeri Kiai Ageng Muhammad Besari Ponorogo
rachsa2223@gmail.com

Abstrak

Kemampuan pemecahan masalah matematis merupakan kompetensi penting dalam pembelajaran matematika. Namun, siswa kelas IX MTsN 2 Ponorogo mengalami kesulitan dalam menyelesaikan soal pemecahan masalah pada materi teorema Pythagoras meliputi pemahaman soal, perencanaan penyelesaian, penguasaan konsep, penarikan kesimpulan, dan pengecekan jawaban. Keterbatasan media pembelajaran konkret juga turut menghambat pemahaman konsep geometris secara visual. Hal ini dipengaruhi oleh berbagai faktor salah satunya adalah model pembelajaran yang diterapkan guru. Pemilihan model pembelajaran yang tepat seperti Model Knisley berbasis Media Tangram, Knisley, dan Direct learning dapat memberikan kontribusi signifikan terhadap peningkatan kemampuan siswa dalam memecahkan masalah matematis. Penelitian ini bertujuan mengetahui perbedaan kemampuan pemecahan masalah matematis siswa: (1) sebelum dan sesudah menerapkan model Direct learning. (2) sebelum dan sesudah menerapkan model Knisley. (3) sebelum dan sesudah menerapkan model Knisley berbasis Tangram. dan (4) antara ketiga model pembelajaran tersebut. Penelitian menggunakan pendekatan kuantitatif dengan desain quasi experimental dan sampel dipilih melalui purposive sampling. Analisis data menggunakan Paired Sample T-Test, One Way ANOVA, dan uji lanjut Tukey HSD. Hasil penelitian menunjukkan: (1) terdapat perbedaan signifikan pada kelas Direct learning (t = -7.009 dan Sig. = 0.000) dengan peningkatan mean 17.39 poin. (2) terdapat perbedaan signifikan pada kelas Knisley (t = -8.883 dan Sig. = 0.000) dengan peningkatan mean 27.96 poin. (3) terdapat perbedaan signifikan pada kelas Knisley berbasis Tangram (t = -7.349 dan Sig.= 0.000) dengan peningkatan mean 29.01 poin. (4) terdapat perbedaan signifikan antar ketiga model (F = 11.655 dan Sig. = 0.000). di mana Knisley berbasis Tangram terbukti paling efektif dibanding Direct learning (Sig. = 0.000) dan Knisley (Sig. = 0.022).
Kesimpulan penelitian ini adalah model pembelajaran Knisley berbasis Media Tangram terbukti paling efektif dalam meningkatkan kemampuan pemecahan masalah matematis siswa dibandingkan dengan model Direct learning dan model Knisley tanpa media. Integrasi media manipulatif konkret ke dalam tahapan pembelajaran Knisley menghasilkan pembelajaran yang lebih bermakna, visual, dan menyenangkan. sehingga mengoptimalkan pemahaman konsep dan kemampuan pemecahan masalah matematis siswa pada materi teorema Pythagoras.

***Kata Kunci**: Model Pembelajaran Knisley, Media Tangram, Kemampuan Pemecahan Masalah Matematis, Teorema Pythagoras*

Abstract

Mathematical problem-solving ability is an essential competency in mathematics learning. However, Grade IX students at MTsN (*Madrasah Tsanawiyah Negeri)* 2 Ponorogo experienced difficulties with mathematical problem-solving tasks on the Pythagorean theorem. encompassing problem comprehension, solution planning. concept mastery, drawing conclusions, and answer verification. The limited availability of concrete learning media further hindered students' visual understanding of geometric concepts. This condition is influenced by various factors, including the teacher's learning model. Selecting an appropriate learning model, such as the Tangram Media based Knisley Model, the Knisley Model, or Direct Learning can significantly improve students' mathematical problem solving ability. This study aimed to determine differences in students' mathematical problem solving ability (1) before and after implementing the Direct Learning model, (2) before and after implementing the Knisley model, (3) before and after implementing the Tangram based Knisley model, and (4) among the three learning models. This study employed a quantitative, quasi-experimental design and purposive sampling. Data analysis was conducted using the Paired Sample T-Test, One-Way ANOVA, and Tukey HSD post hoc test. The results revealed (1) a significant difference in the Direct Learning class (t = –7.009 and Sig. = 0.000) with a mean increase of 17.39 points (2) a significant difference in the Knisley class (t = –8.883 and Sig. = 0.000) with a mean increase of 27.96 points (3) a significant difference in the Tangram based Knisley class (t = –7.349 and Sig. = 0.000) with a mean increase of 29.01 points (4) a significant difference among the three models (F = 11.655 and Sig. = 0.000). in which the Tangram based Knisley model proved significantly more effective than the Direct Learning model (Sig. = 0.000) and the Knisley model (Sig. = 0.022). This study concludes that the Knisley learning model using Tangram media is the most effective in improving students' mathematical problem solving ability compared to the Direct Learning model and the Knisley model without media. The integration of concrete manipulative media into the Knisley learning stages produced more meaningful, visual, and enjoyable learning. thereby optimizing students' conceptual understanding and mathematical problem-solving skills on the Pythagorean theorem.

Keywords : Knisley Learning Model, Tangram Media, Mathematical Problem-Solving Ability, Pythagorean Theorem

INTRODUCTION

Daily human life is inseparable from the vital role of mathematics. Mathematics instruction in education is not merely arithmetic, it also trains students to think logically, instills character values, and equips them with the skills to systematically solve various problems. Mathematics is considered an important tool in developing critical, logical, objective, systematic, rational, and creative thinking patterns that support the development of modern science and technology (Andhini et al. 2023).

Mathematical problem solving skills are one of the key skills students need in mathematics learning. Cognitive processes involving problem understanding. strategy determination, strategy implementation, and evaluation of the results define this skill. According to Sumarmo, mathematical problem solving abilities are essential for solving mathematical problems, problems in other disciplines, and problems in everyday life, which is the vision for developing mathematics learning to meet today's needs (Sumarmo. n.d.).

According to Polya (2004), ideal mathematical problem solving abilities are characterized by four main indicators: students are able to understand and interpret problems correctly, develop a systematic solution plan, apply mathematical concepts correctly, and evaluate and examine the results of problem solving critically and reflectively. Problem solving has three meanings, namely as a goal, as a process, and as a skill (Sumarmo. n.d.). Problem-solving as a goal emphasizes the importance of teaching it. Problem solving as a process can be understood as the activities methods. Strategies or procedures students use to solve problems. Problem-solving as a skill comprises two components general skills that students must have at the school level, and minimum skills that students need to master in the community environment.

Based on initial observations and interviews with mathematics teachers at MTs Negeri 2 Ponorogo. it was found that ninth-grade students experienced difficulty solving mathematical problems. The problems presented were challenging for many students because the instructions were difficult to interpret, and the conclusions drawn varied resulting in inconsistent planning and implementation of the solutions. Students also experienced a lack of mastery of basic concepts, an inability to draw accurate conclusions, and infrequent checking of answers. Students' visual and practical understanding was hampered by the limited availability of concrete teaching aids, resulting in suboptimal problem-solving skills.

This reality was reinforced by the results of the 2022 PISA survey, which showed that Indonesia ranked 68th with an average score of 366. down from 379 in 2018. Research by Eka et al. (2022) also found that 81.25% of students had low problem solving skills. This could be due to students' lack of familiarity with problem solving. leading to many errors in solving problems.

Low mathematical problem-solving ability is inextricably linked to the learning model used. Interviews revealed that teachers still use conventional learning models and have not yet optimally utilized learning media in the classroom. Selecting the right learning model to produce engaging learning and emphasize student activity (student centered learning) is a crucial agenda for teachers. Student centered learning models have been shown to be more effective than conventional teacher-centered models in improving critical thinking, creativity, and problem-solving skills (Puadi & Habibie. 2018).

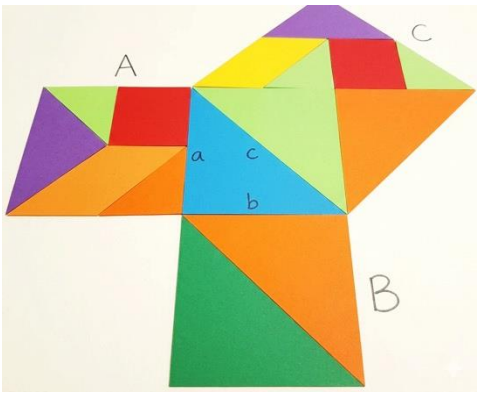
The Knisley learning model is a mathematics learning model developed by Jeff Knisley based on Kolb's experiential learning theory. This model consists of four stages allegorization (analogizing new concepts to existing ones), integration (exploring and distinguishing new concepts from old ones), analysis (connecting concepts logically), and synthesis (applying new concepts to develop more effective problem solving strategies) (Knisley. 2001). This model is student centered, in which students construct their own knowledge through a series of concrete experiences guided in stages.

In addition to selecting the right learning model, teachers need to be creative to make learning more engaging and create a memorable learning environment for students. More interactive and concrete learning media are highly recommended to make the learning process more effective and enjoyable. One learning medium that can be integrated into the Knisley model is the Tangram.

The Tangram is an ancient geometric puzzle from China consisting of seven flat pieces two large triangles, one medium triangle, two small triangles, one square, and one parallelogram. These pieces can be arranged into various geometric shapes. In the context of mathematics learning, specifically the Pythagorean theorem. The Tangram functions as a visual and manipulative tool to concretely prove that $a^2 + b^2 = c^2$. Students arrange the Tangram pieces on the sides of a right triangle and observe that the area of the square on the hypotenuse equals the combined area of the two squares on the sides of the right triangle.

Documentation of the use of Tangram media in this study is shown in Figure 1. It shows an arrangement of colorful Tangram pieces that form squares on the three sides of a right triangle, with labels a (upright side), b (base side), c (hypotenuse), and square labels A, B, and C on each side.

Figure 1. Tangram Media for Visual Proof of the Pythagorean Theorem



In Figure 1. students can directly observe and manipulate the Tangram pieces to visually prove the Pythagorean theorem. Square labeled A is on side a (the perpendicular side), square labeled B is on side b (the base), and square labeled C is on side c (the hypotenuse). By arranging the Tangram pieces, students can observe and verify that area A + area B = area C, which is the Pythagorean theorem statement $a^2 + b^2 = c^2$. This hands on manipulation experience bridges students' abstract understanding of the Pythagorean theorem concept in a concrete and meaningful way. Anwar K. & Jurotun (2019) stated that teaching aids provide students with the opportunity to learn firsthand by touching, observing, and experimenting.

Several previous studies have demonstrated the effectiveness of the Knisley model and Tangram media. Sri Apriantini et al. (2022) conducted a systematic literature review of 32 articles and found that the Knisley model consistently improved students' mathematical abilities from 2012 to 2022. Wirda Rahmani and Nurbaiti Widyasari (in Risma et al. 2024) reported an increase in critical thinking skills in the group taught using Tangram media. compared with the conventional group. Risma Susanti et al. (2024) confirmed that the Knisley model was effective in significantly improving the mathematical problem solving ability of MTs students compared to the control class. Purbasari & Khotimah (2022) proved that Tangram was effective in improving students' cognitive abilities and learning motivation.

Based on the description above, this study aims to (1) determine the difference in mathematical problem solving ability before and after the implementation of the Direct Learning model, (2) determine the difference in mathematical problem-solving ability before and after the implementation of the Knisley model, (3) to find out the differences in mathematical problem solving abilities before and after the application of the Tangram-based Knisley model, and (4) to find out the differences in mathematical problem solving abilities between the three learning models in class IX students at MTsN 2 Ponorogo.

RESEARCH METHOD

This research was conducted at MTsN 2 Ponorogo in the even semester of the 2025/2026 academic year, from January to March 2026. The study used a quantitative approach. Quantitative research methods are defined as methods used to examine a specific population or sample, in which data collection uses research instruments and data analysis is quantitative or statistical, with the aim of testing a predetermined hypothesis (Sugiyono. 2015).

This research type is a comparative study with a quasi experimental design and a non equivalent control group design. Comparative research compares variables between two or more groups (Sugiyono. 2015). This method was chosen because it aligns with the research objective to determine differences in the mathematical problem-solving abilities of students who receive different treatments. The research design used is shown in Table 1 below.

Table 1. Research Design

Class	***Pretest***	***Treatment***	***Posttest***
Knisley	O_1	X_1	O_2
Knisley+Tangram	O_3	X_2	O_4
Direct Learning	O_5	–	O_6

Description: O_1 = Knisley class pretest, X_1 = Knisley treatment, O_2 = Knisley class posttest, O_3 = Knisley + Tangram class pretest, X_2 = Knisley + Tangram treatment, O_4 = Knisley + Tangram class posttest, O_5 = Direct Learning class pretest, O_6 = Direct Learning class posttest.

The study population comprised all 331 ninth-grade students at MTsN 2 Ponorogo. across 12 classes. The sampling technique used was a non probability purposive sampling, namely, sampling based on certain considerations. The criteria used were homogeneity of initial class conditions and similarity of teachers. Based on the subject teachers' suggestions, the three classes that met these criteria were classes IX C, IX D, and IX H. Class IX C (28 students) was designated as the Direct Learning class, class IX H (28 students) as the Knisley class, and class IX D (28 students) as the Tangram based Knisley class with a total sample of 84 students.

The dependent variable (Y) in this study is mathematical problem solving ability. measured based on four Polya indicators (1) understanding the problem students can understand the conditions of the problem or existing problem, (2) making a plan students can think of important steps and formulate the problem into a mathematical model, (3) implementing the plan. students can carry out calculations with the chosen method or formula, and (4) checking back students are able to check the results obtained to prove the correctness of the answer.

The independent variable (X) is a learning model consisting of three categories (1) Direct Learning. a direct learning model with five stages orientation, presentation, structured practice, guided practice, and independent practice, (2) Knisley a learning model with four stages allegorization, integration, analysis, and synthesis, (3) Knisley based on Tangram Media a Knisley model integrated with Tangram manipulative media at each stage.

The data collection instrument was a mathematical problem solving test in the form of descriptive questions on the Pythagorean theorem, comprising a pretest and a posttest. Each test contained 3 questions with cognitive levels C4 (analyzing) and C5 (evaluating) according to the revised Bloom's taxonomy. The instrument was validated by a mathematics education lecturer at UIN Kiai Ageng Muhammad Besari Ponorogo. and a mathematics subject teacher at MTsN 2 Ponorogo. After being declared feasible the instrument was piloted in classes outside the sample. A summary of the instrument trial results is presented in Table 2.

Table 2. Summary of Test Instrument Trial Results

Question Number	***Validity***	***Reliability***	***Difficulty Level***	Description
1	*Valid*	*0.812*	*Medium*	*Can be used*
2	*Valid*	*0.812*	*Medium*	*Can be used*
3	*Valid*	*0.812*	*Medium*	*Can be used*

The data analysis techniques used include (1) normality test using Kolmogorov-Smirnov to determine whether the data is normally distributed, (2) homogeneity test using Levene's Test to determine whether the data variance is homogeneous, (3) Paired Sample T-Test to test differences in mathematical problem-solving ability before and after treatment in each class, (4) One Way ANOVA to test differences in mathematical problem-solving ability between the three models simultaneously, and (5) Tukey HSD further test to identify pairs of models that differ significantly. All tests were conducted using IBM SPSS Statistics 25 software with a significance level of $\alpha = 0.05$.

The categorization of students' mathematical problem solving ability was determined using the mean (Mx) and standard deviation (SDx) formula as follows (a) scores greater than Mx + 1.SDx are categorized as high, (b) scores less than Mx – 1.SDx are categorized as low, (c) scores between Mx – 1.SDx and Mx + 1.SDx are categorized as moderate (Lestari & Yudhanegara. 2015).

Research Data Description

The research was conducted at MTsN 2 Ponorogo from January 20 to March 14 2026 involving three sample classes totaling 84 students. Data on mathematical problem solving ability were collected through a pre-test before the treatment and a post test after the treatment. Descriptive statistics for the three classes are presented in Table 3.

Table 3. Descriptive Statistics of Pretest and Posttest

Class / Model	*Mean Pre*	*SD Pre*	*Mean Post*	*SD Post*	*Increased Number*
Direct Learning	*54.04*	*12.00*	*71.43*	*9.00*	*+17.39*
Knisley	*49.43*	*11.00*	*77.39*	*9.00*	*+27.96*
Knisley+Tangram	*56.11*	*13.00*	*85.12*	*10.00*	*+29.01*

Based on Table 3. all three classes experienced a mean increase from pretest to posttest. The Tangram based Knisley class showed the highest posttest mean (85.12) and the largest increase (+29.01 points), followed by Knisley (+27.96 points) and Direct Learning (+17.39 points). The description of the mathematical problem solving ability categories of Direct Learning class students before treatment is shown in Table 4.

Table 4. Categories of Mathematical Problem Solving Ability Direct Learning Class Pretest

Number	*Score*	*Frequency*	*Percentage*	*Criterion*
1	*> 66*	*6*	*21%*	*High*
2	*42–66*	*19*	*68%*	*Medium*
3	*< 42*	*3*	*11%*	*Low*
	Total Score	*28*	*100%*	

Table 4 shows that before the Direct Learning model was implemented, 21% of students were in the high category, 68% were in the medium category, and 11% were in the low category. Thus. the initial mathematical problem solving ability of the Direct Learning class was generally in the medium category.

Table 5 shows that after implementing the Direct Learning model, 11% of students were in the high category, 78% in the medium category, and 11% in the low category. There was a slight increase in the medium category, but the proportion of the high category remained small.

Table 5. Categories of Mathematical Problem Solving Ability Direct Learning Class Posttest

Number	*Score*	*Frequency*	*Percentage*	*Criterion*
1	*> 80*	*3*	*11%*	*High*
2	*62–80*	*22*	*78%*	*Medium*
3	*< 62*	*3*	*11%*	*Low*
	Total Score	*28*	*100%*	

Table 6. Categories of Mathematical Problem Solving Ability Knisley Class Pretest

Number	*Score*	*Frequency*	*Percentage*	*Criterion*
1	*> 60*	*6*	*21%*	*High*
2	*38–60*	*16*	*57%*	*Medium*
3	*< 38*	*6*	*21%*	*Low*
	Total Score	*28*	*100%*	

Table 6 shows that before the Knisley model treatment, 21% of students were in the high category, 57% in the medium category, and 21% in the low category. This distribution is relatively more even than in the Direct Learning class.

Table 7. Categories of Mathematical Problem Solving Ability Knisley Class Posttest

Number	*Score*	*Frequency*	*Percentage*	*Criterion*
1	*> 86*	*6*	*21%*	*High*
2	*68–86*	*19*	*68%*	*Medium*
3	*< 68*	*3*	*11%*	*Low*
	Total Score	*28*	*100%*	

Table 7 shows a significant increase 21% of students were in the high category (up from 21%), 68% were in the medium category, and only 11% were in the low category (down from 21%). indicating the effectiveness of the Knisley model in improving mathematical problem solving ability.

Table 8 shows that after implementing the Knisley Model Based on Tangram Media 11% of students were in the high category, 78% in the medium category, and 11% in the low category. There was a slight increase in the medium category, but the proportion of the high category remained small.

Table 8. Mathematical Problem Solving Ability Categories Knisley+Tangram Class Pretest

Number	*Score*	*Frequency*	*Percentage*	*Criterion*
1	*> 69*	*3*	*11%*	*High*
2	*43–69*	*22*	*78%*	*Medium*
3	*< 43*	*3*	*11%*	*Low*
	Total Score	*28*	*100%*	

Table 9. Mathematical Problem Solving Ability Categories Knisley+Tangram Class Posttest

Number	*Score*	*Frequency*	*Percentage*	*Criterion*
1	*> 95*	*7*	*25%*	*High*
2	*75–95*	*19*	*68%*	*Medium*
3	*< 75*	*2*	*7%*	*Low*
	Total Score	*28*	*25%*	

Table 9 shows the most impressive results 25% of students were in the high category (up from 11%), 68% were in the medium category, and only 7% were in the low category (down from 11%). The Tangram-based Knisley model was most effective in encouraging students to reach the high category.

Prerequisite Test Results

Before conducting the hypothesis test, a prerequisite test was first conducted. The results of the Kolmogorov Smirnov normality test for the pretest and posttest data for all classes are presented in Table 10.

Table 10. Results of the Kolmogorov-Smirnov Normality Test

Class / Data	*D max*	*D tabel*	*Sig.*	*Decision*
Pretest DL	*0.152*	*0.250*	*0.082*	*normally distributed*
Pretest Knisley	*0.138*	*0.250*	*0.183*	*normally distributed*
Pretest K+T	*0.143*	*0.250*	*0.149*	*normally distributed*
Posttest DL	*0.092*	*0.250*	*0.200*	*normally distributed*
Posttest Knisley	*0.112*	*0.250*	*0.200*	*normally distributed*
Posttest K+T	*0.087*	*0.250*	*0.200*	*normally distributed*

Based on Table 10. the Dmax value is < D table, and the significance value is> 0.05 for all pretest and posttest data, so it can be concluded that all data are normally distributed. Furthermore the results of Levene's test of homogeneity are presented in Table 11.

Table 11. Results of Levene's Test of Homogeneity

Data	*F Value*	*F table*	*Sig.*	*Decision*
Pretest (3 classes)	*1.124*	*3.110*	*0.330*	*homogeneous variance*
Posttest (3 classes)	*1.348*	*3.110*	*0.265*	*homogeneous variance*

Table 11 shows that the calculated F value is < the F table value and the significance is> 0.05 for both the pretest and posttest data, indicating that the variance across the three groups is homogeneous. With these two prerequisites met, the analysis can be continued using parametric tests.

Results of Hypothesis Testing 1, 2, and 3: Paired Sample T-Test

A paired samples t-test was used to examine differences in mathematical problem solving ability before and after treatment within each class. The hypotheses tested were H_0: there is no difference in mathematical problem solving ability before and after treatment, and H_1: there is a difference in mathematical problem solving ability before and after treatment. The test results are presented in Table 12.

Table 12. Paired Sample T-Test Results

Class	*t Value*	*t table*	*Sig.*	*Decision*
Direct Learning	*-7.009*	*2.052*	*0.000*	*Rejecting H_0*
Knisley	*-8.883*	*2.052*	*0.000*	*Rejecting H_0*
Knisley+Tangram	*-7.349*	*2.052*	*0.000*	*Rejecting H_0*

Based on Table 12, the three classes show a value of |t count| > t table and a significance of 0.000 < 0.05, so that H_0 is rejected and H_1 is accepted. It can be concluded that: (1) there is a significant difference in mathematical problem solving ability before and after the application of the Direct Learning model, (2) there is a significant difference in mathematical problem solving ability before and after the application of the Knisley model, (3) there is a significant difference in mathematical problem solving ability before and after the application of the Tangram-based Knisley model.

Hypothesis Test Results 4: One-Way ANOVA and Tukey HSD

One-Way ANOVA was used to simultaneously examine differences in mathematical problem solving ability between the three learning model groups. The hypotheses tested were H_0: there is no difference in mathematical problem solving ability between the three models, and H_1: there is a difference in mathematical problem solving ability between the three models. The test results are presented in Table 13.

Table 13. One-Way ANOVA Test Results

Variation Sources	*Sum of Squares*	*Df*	*Mean Square*	*F*	*Sig.*
Between Groups	*3.224.60*	*2*	*1.612.30*	*11.655*	*0.000*
Within Groups	*11.190.75*	*81*	*138.28*		
Total	*14.415.35*	*83*			

Table 13 shows F Value = 11.655 with Sig. = 0.000 < 0.05. so H_0 is rejected. There is a significant difference in mathematical problem solving ability between the three learning models. To identify the different model pairs, a further Tukey HSD test was conducted in Table 14.

Table 14. Tukey HSD Post Hoc Test Results

Model Comparison	*Mean*	*Sig.*	*Decision*
Direct Learning vs Knisley	*5.96*	*0.097*	*There are no significant differences*
Direct Learning vs Knisley+Tangram	*13.69*	*0.000*	*There are significant differences*
Knisley vs Knisley+Tangram	*7.73*	*0.022*	*There are significant differences*

Table 14 shows that the Tangram-based Knisley model differs significantly from Direct Learning (Sig. = 0.000) and from Knisley (Sig. = 0.022). Meanwhile, Direct Learning and Knisley are not significantly different (Sig. = 0.097). Thus, the Tangram-based Knisley model is statistically superior.

Discussion

Differences in Mathematical Problem-Solving Ability in Direct Learning Classes

Based on the results of the Paired Sample T-Test in the Direct Learning class, where 'Direct Learning' is defined as a structured, teacher-led instructional approach, the t-value was -7.009 with a significance level of 0.000 ($p < 0.05$). The mean score for 'Mathematical Problem-Solving Ability' the students' capacity to analyze and solve math problems increased from 54.04 in the pretest to 71.43 in the posttest, a 17.39-point increase. This indicates that the Direct Learning model significantly improved students' Mathematical Problem-Solving Ability.

Furthermore, this finding aligns with the characteristics of the Direct Learning model proposed by Arends. According to Arends, this model is designed to support learning processes related to structured declarative and procedural knowledge. Specifically, the five stages of orientation, presentation, structured practice, guided practice, and independent practice provide a clear framework for students in learning problem-solving procedures.

However, the posttest distribution shows that 78% of students remain in the medium category, while only 11% reach the high category. This indicates that although Direct Learning is effective in improving basic skills, this model is less than optimal in developing mathematical problem-solving abilities to a higher level, given its teacher-centered nature and minimal independent exploration by students. This finding is consistent with the results of research by Puta Linda Apsari et al., which shows that Direct Learning is less effective than the Tangram-assisted model.

Differences in Mathematical Problem-Solving Ability in the Knisley Class

The results of the Paired Sample T-Test in the Knisley class yielded $t = -8.883$ (Sig. = 0.000), indicating a significant difference between the pretest (mean = 49.43) and posttest (mean = 77.39), with an increase of 27.96 points. The percentage of students in the low category decreased from 21% to 11%, indicating that the Knisley model is effective at moving low-ability students into the medium and high categories.

The success of the Knisley model aligns with its theoretical foundation. In the allegorization stage, students are introduced to the Pythagorean theorem through connections with familiar concepts. In the integration stage, students compare new concepts with old ones through active exploration. In the analysis stage, students solve problems logically step by step. In the synthesis stage, students utilize concepts as tools to solve more complex problems (Knisley, 2001). Knisley emphasizes that, in this model, students construct their own knowledge, aligning with the constructivist perspective of Brooks and Brooks.

This finding confirms the research of Risma Susanti et al. (2024) that classes implementing the Knisley model at the MTs level achieved significantly higher mathematical problem-solving ability scores than control classes. This also supports the Systematic Literature Review by Apriantni et al. (2022), which demonstrated the consistent effectiveness of the Knisley model on various students' mathematical abilities.

Differences in Mathematical Problem-Solving Ability in Tangram-Based Knisley Class

The Tangram-based Knisley model implemented demonstrated the highest effectiveness. The t-value was -7.349 (Sig. = 0.000). The mean posttest score reached 85.12, an increase of 29.01 points from the pretest of 56.11. The most significant improvement was seen in the distribution of the high category, which rose from 11% to 25%.

As shown in Figure 1, integrating Tangram media into the Knisley stages enabled students to arrange Tangram pieces on the sides of a right triangle to concretely prove that $a^2 + b^2 = c^2$. In the allegorization stage, students analogized the Tangram pieces with the familiar concept of area. In the integration stage, students explored and arranged Tangram pieces on sides a, b, and c. In the analysis stage, students observed and analyzed the relationship between the areas of the three squares. In the synthesis stage, Students apply this understanding to solve more complex Pythagorean theorem problems.

Anwar K. & Jurotun (2019) stated that teaching aids provide students with the opportunity to learn firsthand through touch, observation, and experimentation. This aligns with Tangram Media's mission to make learning more interactive, visual, and meaningful. This concrete and engaging learning experience also increases students' interest and motivation throughout the learning process, thereby improving mathematical problem-solving skills.

Comparison of Three Learning Models

A one-way ANOVA test yielded F = 11.655 (Sig. = 0.000 < 0.05), confirming significant differences between the three models. A further Tukey HSD test showed that the Tangram based Knisley model was statistically superior: significantly different from both Direct Learning (Sig. = 0.000) and Knisley (Sig. = 0.022). Meanwhile, Direct Learning and Knisley did not differ significantly (Sig. = 0.097), although Knisley's mean was higher.

Polya (2004) states that the four steps of effective problem solving include understanding the problem, developing a plan, implementing the plan, and checking the answer. These four indicators were measured in this study. The results indicate that the Tangram based Knisley model optimally facilitates students through all these stages. Tangram media helps students understand problems visually (as shown in Figure 1), while the Knisley stages guide students from concrete exploration to the application of concepts in complex problem solving.

Joyce and Weil asserted that selecting the right learning model can develop students' thinking and improve their problem solving abilities. The Tangram-based Knisley model proves this statement because the combination of concrete media and systematic learning stages results in the most optimal increase in mathematical problem-solving abilities. This finding is in line with research by Nadia Pramitasuri et

al., which proves that the use of concrete media significantly improves mathematical problem-solving abilities.

CONCLUSION

Based on the research and discussion conducted at MTsN 2 Ponorogo, the following conclusions can be drawn. First, there is a significant difference in mathematical problem solving ability between before and after implementing the Direct Learning model in grade IX students of MTsN 2 Ponorogo (t = -7.009 and Sig. = 0.000), with an average score increase of 17.39 points from 54.04 to 71.43. Second, there is a significant difference in mathematical problem solving ability between before and after implementing the Knisley learning model in grade IX students of MTsN 2 Ponorogo (t = -8.883 and Sig. = 0.000), with an average score increase of 27.96 points from 49.43 to 77.39. Third, there is a significant difference in mathematical problem solving ability between before and after applying the Tangram based Knisley learning model in grade IX students of MTsN 2 Ponorogo (t = -7.349 and Sig. = 0.000). With an increase in the average value of 29.01 points from 56.11 to 85.12, the highest among the three models. Fourth, there is a significant difference in mathematical problem solving ability between the three learning models (F = 11.655 and Sig. = 0.000). The Tukey HSD further test shows that the Tangram based Knisley model is significantly more effective than the Direct Learning model (Sig. = 0.000) and the Knisley model (Sig. = 0.022). Thus, the Tangram based Knisley model has proven to be the most effective learning model in improving students' mathematical problem-solving abilities on the Pythagorean theorem in grade IX MTsN 2 Ponorogo.

SUGGESTIONS

Based on the research results, the researcher proposes the following suggestions (1) For mathematics teachers, it is recommended to use the Tangram-based Knisley model as an alternative in mathematics learning activities, especially on geometry material, considering that this model has proven to be the most superior in improving students' mathematical problem-solving abilities, (2) For students, it is recommended to be more active and enthusiastic in participating in innovative learning and to utilize the learning media provided by the teacher optimally, (3) For further researchers, it is recommended to expand the application of the Tangram-based Knisley model to other mathematics materials or different levels of education, as well as explore other variables that may be influenced by this learning model, such as mathematical communication skills, creative thinking, or self-efficacy.

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